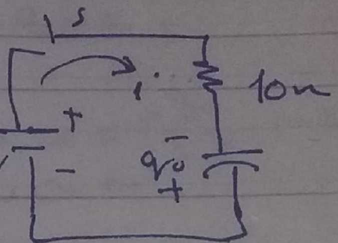
Sec (11)

Laplace + Transient

3/5 - 5/5

Sheet(9)

[1] in RC Series Circuit, capacitor is initially charged with $q = 2500 \times 10^{-6}$ Col., at $t=0$ switch closed, a constant supply 100V applied to the circuit. find the current using Laplace.



Sol

$$V = Ri + \frac{1}{C}\int i\,dt$$

$$100 = 10\,i + \frac{1}{50\times10^{-6}}\int i\,dt \rightarrow (1)$$

Taking laplace transform of previous equation

∴ S-domain → initial condition

$$\frac{100}{S} = 10\,I(S) + \frac{I(S)}{50\times10^{-6}S} + \frac{q_0}{50\times10^{-6}S}$$

$$\therefore I(S) = \left[\frac{100}{S} + \frac{2500\times10^{-6}}{50\times10^{-6}S}\right] \Big/ \left[10 + \frac{1}{50\times10^{-6}S}\right]$$

$$I(S) = \{150/S\} \Big/ \left[10 + \frac{2\times10^{4}}{S}\right]$$

$$\boxed{I(S) = \frac{150}{10S + 2\times10^{4}} = \frac{15}{S + 2\times10^{3}}}$$

Take inverse laplace $L^{-1}(I(S))$

$$I(t) = 15\,e^{-2\times10^{3}t}$$

مهم يحفظ

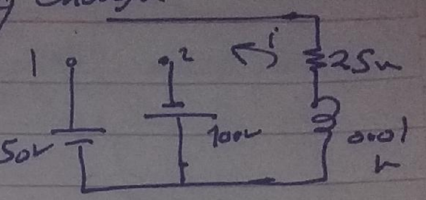
$$A e^{-at} \leftarrow \frac{A}{S+a}$$

$$A \leftarrow A/s$$

Note $\int i\,dt = \frac{I(s)}{S} + \frac{q(0^+)}{S}$ (initial)

$\cancel{\int i}$, $i = \frac{dq}{dt}$ ∴ $\cancel{\int i = q}$

$\boxed{2}$ at RL Circuit, S is in position 1 long enough to establish steady state condition and at $t=0$ switched to Pos 2. Find I.



sol

$$100 = 25i + (0.01)\, di/dt \rightarrow (1) \text{ at Pos 2}$$

Note at Pos 1 $\rightarrow$ steady state So $i = -\frac{50}{25} = -2A$

عن ما يأتي المرسوم عكس الإتجاه

$\mathcal{L}\frac{di}{dt} = S I(S) - I(0^+)$

Taking laplace of (1)

$$\therefore \quad \frac{100}{S} = 25 I(S) + [0.01\, I(S)]S - 0.01\, I(0^+)$$

$$\therefore \quad \frac{100}{S} = 25 I(S) + 0.01 S\, I(S) - 0.01 \times (-2)$$

$$I(S) = \frac{100}{S(0.01S+25)} - \frac{2\times 0.01}{(0.01S+25)} = \cancel{\frac{10^4}{S(S+2500)} - \frac{2}{S+2500}}$$

$$I(S) = \frac{10^4}{S(S+2500)} - \frac{2}{S+2500} = \frac{A}{S} + \frac{B}{S+2500} - \frac{2}{S+2500}$$

$$\text{Now} \quad \frac{10^4}{S(S+2500)} = \frac{A}{S} + \frac{B}{S+2500}$$

or $A(S+2500) + B(S) = 10^4$

$AS + BS = 0 \longrightarrow A + B = 0$

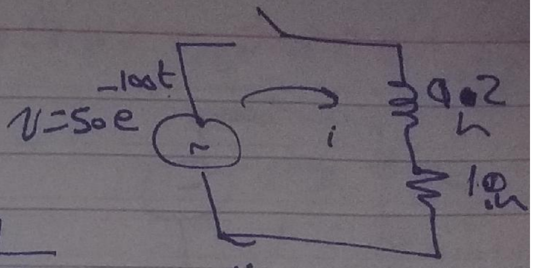
$A(2500) = 10^4 \longrightarrow A = 4, \ B = -4$

$$\therefore I = \frac{4}{S} - \frac{4}{S+2500} - \frac{2}{S+2500} = \frac{4}{S} - \frac{6}{S+2500}$$

$$\therefore I(t) = 4 - 6e^{-2500t}$$

[3] Find i

$$50e^{-100t} = 10i + 0.2\frac{di}{dt}$$



Take laplace $\quad Ae^{-at} \rightarrow \frac{A}{s+a}$

$\frac{di}{dt} = sI(s) - i(0^+)$

$$\frac{50}{s+100} = 10I(s) + 0.2sI(s) - 0.2i(0^+)$$

$\rightarrow 0$ at $t=0 \rightarrow i=$

$$\therefore \frac{50}{s+100} = I(s)\left[10 + 0.2s\right]$$

$$I(s) = \frac{50}{(s+100)(0.2s+10)} = \frac{50}{(s+100)(s+50)0.2}$$

$$I(s) = \frac{250}{(s+100)(s+50)} = \frac{A}{s+100} + \frac{B}{s+50}$$

$$\therefore A(s+50) + B(s+100) = 250$$

$\therefore sA + Bs = 0$ or $\underline{A = -B}$

$50A + 100B = 250$

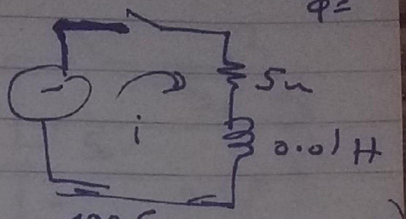
or $50B = 250$, $B = 5$, $A = -5$

$$\therefore I(s) = \frac{-5}{s+100} + \frac{5}{s+50}$$

$$\therefore I(t) = -5e^{-100t} + 5e^{-50t}$$

(4) series RL (g $V = 100\sin(500t+\phi)$, find I if S closed at $\phi = 0$

Sol



$$100\sin(500t+\phi) = 5i + 0.01\frac{di}{dt}$$

- take laplace -

$$5I(S) + 0.01 S I(s) - 0.01\, i(0^+) = \frac{100\left(\cancel{100\times 500}\, S\sin\phi + 500\cos\phi\right)}{S^2+(500)^2}$$

$$5I(S) + 0.01S = 0.01 \times \text{zero} + \frac{(100)(500)}{S^2+(500)^2}$$

$$I(S)\left[0.01S+5\right] = \frac{5\times10^4}{S^2+(500)^2}$$

$$I(S) = \frac{5\times10^4}{\left((0.01)S+5\right)(S^2+500^2)} = \frac{5\times10^6}{(S+500)(S^2+500^2)}$$

$$I(S) = \frac{5\times10^6}{(S+500)(S+j500)(S-j500)} = \frac{A}{S+500} + \frac{B}{S+j500} + \frac{C}{S-j500}$$

$a_1 = -500$, $a_2 = -j500$, $a_3 = j500$

$$i(t) = \frac{P(a_1)}{Q'(a_1)}e^{-500t} + \frac{P(a_2)}{Q'(a_2)}e^{-j500t} + \frac{P(a_3)}{Q'(a_3)}e^{j500t}$$

where $P(a_1) = P(a_2) = P(a_3) = 5\times10^6$

$Q(S) = S^3 + 5S^2 + (500)^2 S + (500)^2 \times 5$

$Q'(S) = 3S^2 + 10S + (500)^2$

$Q'(a_1) = 3(-500)^2 - 10\times500 + (500)^2 =$ ✓

$Q'(a_2) = 3(-(500)^2) + (10\times -j500) + (500)^2 =$ ✓

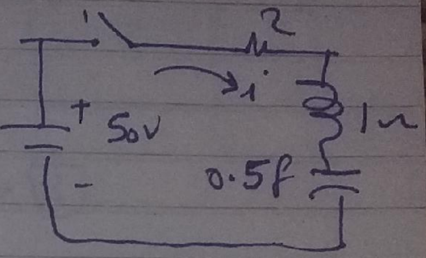
$Q'(a_3) = -3(500)^2 + 10\times j500 + (500)^2 =$ ✓

$$i(t) = 10\sin 500t - 10\cos 500t + 10e^{500t}$$

OR By Partial Fraction → $i = 5\left(\frac{-1+j}{S+j500}\right) + 5\left(\frac{-1-j}{S+500}\right) + \frac{10}{S+500}$

5) in series RLC, no initial charge on cap, at $t=0$ close S, find i

Sol



$$50 = 2i + 1\frac{di}{dt} + \frac{1}{0.5}\int i\,dt$$

Take laplace

$$\frac{50}{S} = 2I(S) + [SI(S) - 1 \times \cancel{i(0^+)}] + \frac{1}{0.5}\left[\frac{I(S)}{S} + \cancel{\frac{\dots}{S}}\right]$$

$$\frac{50}{S} = 2I(S) + SI(S) + \frac{2}{S}I(S)$$

$$\frac{50}{S} = I(S)\left[2 + S + \frac{2}{S}\right]$$

$$50 = I(S)[2S + S^2 + 2]$$

$$I(S) = \frac{50}{S^2 + 2S + 2} = \frac{50}{(S+1+j)(S+1-j)}$$

$$I(S) = \frac{A}{S+1+j} + \frac{B}{S+1-j}$$

By partial Fraction

$A = j25$, $B = -j25$

$$\therefore I(S) = \frac{j25}{S+1+j} - \frac{j25}{S+1-j}$$

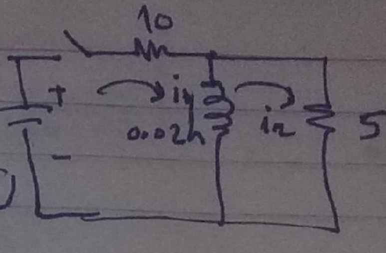
$$i = j25\, e^{-(1+j)t} - j25\, e^{-(1-j)t}$$

$$= 50\, e^{-t} \sin t$$

where $e^{jx} = \cos x + j\sin x$

$\boxed{6}$ in two mesh find currents i_1, i_2

Sol



$$10 i_1 + 0.02 \frac{di_1}{dt} - 0.02 \frac{di_2}{dt} = 100 \rightarrow \boxed{1}$$

$$0.02 \frac{di_2}{dt} - 0.02 \frac{di_1}{dt} + 5 i_2 = 0 \rightarrow \boxed{2}$$

Take laplace for 2 equations

$$10 I_1(s) + 0.02[s I_1(s) - \cancel{i_1(0^+)}] - 0.02[s I_2(s) - \cancel{i_2(0^+)}] = \frac{100}{s}$$

$$\therefore I_1(s)[10 + 0.02s] - I_2(s)[0.02s] = \frac{100}{s} \rightarrow 1$$

or $5 I_2(s) + 0.02[s I_2(s) - \cancel{i_2(0^+)}] - 0.02[s I_1(s) - \cancel{i_1(0^+)}] = 0$

$$\therefore I_1(s)[-0.02s] + I_2(s)[5 + 0.02s] = 0 \rightarrow 2$$

$\therefore$ From eq (2)

$$I_2(s) = \frac{+0.02s \; I_1(s)}{5 + 0.02s} = \frac{+s \; I_1(s)}{\frac{5}{0.02} + s}$$

$$I_2(s) = I_1(s) \frac{s}{s + 250} \rightarrow 3$$

From (1)

$$I_1(s)[10 + 0.02s] - \left(\frac{s}{s+250}\right) \times (0.02s) \, I_1(s) = \frac{100}{s}$$

$$I_1(s)\left[10 + 0.02s - \frac{0.02s^2}{s+250}\right] = \frac{100}{s}$$

$$I_1(s)\left[\frac{2500 + 10s + \cancel{0.02s^2} + 5s - \cancel{0.02s^2}}{s+250}\right] = \frac{100}{s}$$

$$I_1(s) = \frac{25000 + 100s}{s(15s + 2500)} = \frac{\frac{1}{15} \times 100 (s + 250)}{s(s + 166.7)}$$

$$\therefore\ I_1(S) = 6.667 \left\{ \frac{S+250}{S(S+166.7)} \right\}$$

$$= \frac{6.67\cancel{S}}{\cancel{S}(S+166.7)} + \frac{6.67 \times 250}{S(S+166.7)}$$

$$I_1(S) = \frac{6.67}{S+166.7} + \frac{A}{S} + \frac{B}{S+166.7}$$

$$A(S+166.7) + BS = 6.67 \times 250$$

$$A(166.7) = 6.67 \times 250 \quad ; \quad \underline{A = 10}$$

$$AS + BS = 0 \quad \text{or } A = -B \quad \therefore \underline{B = -10}$$

$$\therefore I_1(S) = \frac{6.67}{S+166.7} + \frac{10}{S} - \frac{10}{S+166.7}$$

$$\text{or } I_1(S) = \frac{10}{S} - \frac{3.33}{S+166.7}$$

$$\boxed{i_1 = 10 - 3.33\, e^{-166.7t}}$$

$$I_2(S) = I_1(S) \frac{S}{S+250} = 6.67 \frac{\cancel{S+250}}{S(S+166.7)} \frac{\cancel{S}}{\cancel{S+250}}$$

$$I_2(S) = \frac{6.67}{S+166.7}$$

$$\therefore I_2(t) = 6.67\, e^{-166.7t}$$

خلاصه القوانين (التحويلات)

	$f(t)$		Laplace $f(S)$
1	A	$t \geq 0$	A/S
2	At		A/S^2
3	e^{-at}		$\frac{1}{S+a}$
4	te^{-at}		$\frac{1}{(S+a)^2}$
5	$\sin\omega t$		$\frac{\omega}{S^2+\omega^2}$
6	$\cos\omega t$		$\frac{S}{S^2+\omega^2}$
7	$\sin(\omega t+\phi)$		$\frac{S\sin\phi+\omega\cos\phi}{S^2+\omega^2}$
8	$\cos(\omega t+\phi)$		$\frac{S\cos\phi-\omega\sin\phi}{S^2+\omega^2}$
9	$\frac{di}{dt}$		$SI(S)-i(0^+)$
10	$\int i\,dt$		$\frac{I(S)}{S}+\frac{V_{c\ initial}}{S}$ → q/c
11			